

Week 2 Problems

Posted: Sep. 10, 2016

Deadline: Oct. 1, 2016, 11:59PM

Show complete solutions and explanations.

4. An urn contains 20 balls colored either red, white, or black. If the number of black balls in the urn is doubled, the probability of drawing a red ball becomes $1/25$ less than the probability of drawing a red ball in the beginning. If we remove all red balls, the probability of drawing a black ball becomes $1/16$ more than the probability of drawing a black ball in the beginning. How many white balls are there?
5. Find the largest positive integer $n < 2016$ for which the sum of the reciprocals of the non-zero digits of the integers from 1 to 10^n (inclusive) is also an integer.
6. A square $ABCD$ and two points E and F outside of this square are given so that the triangles BEC and CFD are equilateral. Prove that the triangle AEF is also equilateral.
7. Let ABC be an acute-angled triangle with altitude AD , and let M be any point on AD . Denote by E the intersection of the extension of BM with AC , and F the intersection of the extension of CM with AB . Prove that $\angle ADE = \angle ADF$.

8. If $x + y + z = 0$, prove that

$$\frac{x^2 + y^2 + z^2}{2} \cdot \frac{x^5 + y^5 + z^5}{5} = \frac{x^7 + y^7 + z^7}{7}.$$

9. Determine the number of ordered pairs of positive integers (x, y) which satisfy the equation

$$\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}} = \frac{1}{\sqrt{2016}}$$

About the PEM Weekly Problems

The PEM Weekly Problems aims to challenge and enrich high school students' creativity and critical thinking skills by exposing them to non-routine math problems and puzzles. While the problem sets are primarily intended for PEM participants, everyone is encouraged to submit their solutions to us. We acknowledge on the page everyone who submits correct answers. Moreover, PEM participants who solve the most number of problems will be recognized and awarded during the PEM closing ceremony.

Submitting Solutions

1. Typeset and handwritten solutions are welcome. For handwritten solutions, please scan or take a clear photo of your paper.
2. Indicate in the submission your name, school, and year level.
3. Send your solution to ateneo.tuklas@gmail.com.