

Week 3 Problems

Posted: Oct. 2, 2016

Deadline: Oct. 9, 2016, 11:59PM

Show complete solutions and explanations.

10. Find the exact value of
- x
- , where

$$x = \frac{\sum_{n=1}^{44} \sin n^\circ}{\sum_{n=1}^{44} \cos n^\circ}$$

11. Let X and Y be two points which lie on the arc BC of the circumcircle of $\triangle ABC$ such that $\angle BAX = \angle CAY$. If M is the midpoint of the chord AX , prove that $BM + CM > AY$.
12. Let A be a set containing 2016 real numbers. Define

$$S = \{x + y \mid x, y \in A\}, D = \{x - y \mid x, y \in A\}.$$

Prove that $|A| \cdot |D| \leq |S|^2$. (Here, $|X|$ refers to the cardinality of the set X .)

About the PEM Weekly Problems

The PEM Weekly Problems aims to challenge and enrich high school students' creativity and critical thinking skills by exposing them to non-routine math problems and puzzles. While the problem sets are primarily intended for PEM participants, everyone is encouraged to submit their solutions to us. We acknowledge on the page everyone who submits correct answers. Moreover, PEM participants who solve the most number of problems will be recognized and awarded during the PEM closing ceremony.

Submitting Solutions

1. Typeset and handwritten solutions are welcome. For handwritten solutions, please scan or take a clear photo of your paper.
2. Indicate in the submission your name, school, and year level.
3. Send your solution to ateneo.tuklas@gmail.com.