PROGRAM OF EXCELLENCE IN MATHEMATICS

Weekly Problems

Week 4 Problems

Posted: Oct. 9, 2016 **Deadline: Oct. 16, 2016, 11:59PM**

Show complete solutions and explanations.

13. Suppose that points A and C lie on the circumference of a circle with radius $\sqrt{50}$. Let B be a point inside the circle such that $\angle ABC = 90°$. If $AB = 6$ and $BC = 2$, find the distance from B to the center of the the circle.

14. For how many ordered pairs (a, b) of integers does the system

$$\begin{cases} ax + by = 1 \\ x^2 + y^2 = 50 \end{cases}$$

have at least one solution, and at the same time, all of these solutions are ordered pairs (x, y) of integers?

15. A sequence of numbers, beginning with the number 1, is obtained by applying in each step, one of the following two operations:

O1: Multiply the previous number by 2.

O2: Shuffle the digits of the previous number, but without allowing the number to begin with the digit 0.

For example, the number 1000 can be an element of such a sequence, since applying O1 nine times to the number 1 gives 512. Then by O2, we can obtain 125 from 512. Applying O1 to 125 three times then gives 1000.

Is it possible for the number 9,876,543,210 to appear in this sequence?

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About the PEM Weekly Problems

The PEM Weekly Problems aims to challenge and enrich high school students' creativity and critical thinking skills by exposing them to non-routine math problems and puzzles. While the problem sets are primarily intended for PEM participants, everyone is encouraged to submit their solutions to us. We acknowledge on the page everyone who submits correct answers. Moreover, PEM participants who solve the most number of problems will be recognized and awarded during the PEM closing ceremony.

Submitting Solutions

1. Typeset and handwritten solutions are welcome. For handwritten solutions, please scan or take a clear photo of your paper.
2. Indicate in the submission your name, school, and year level.
3. Send your solution to `ateneo.tuklas@gmail.com`.