

## Week 5 Problems

Posted: Oct. 16, 2016

Deadline: Oct. 22, 2016, 11:59PM

*Show complete solutions and explanations.*

16. Let  $a$  and  $b$  be real numbers such that  $\frac{a^2}{1+a^2} + \frac{b^2}{1+b^2} = 1$ . Determine all possible values of the expression

$$(a + b) \left( \frac{a}{1 + a^2} + \frac{b}{1 + b^2} \right).$$

17. Let  $m$  and  $n$  be integers such that  $1 \leq m \leq n$ . Prove that  $m$  divides the number

$$n \sum_{k=0}^{m-1} (-1)^k \binom{n}{k}.$$

18. Let  $A$  be a subset of the set  $\{1, 2, 3, \dots, 15\}$  such that no three distinct integers in  $A$  have a product which is a perfect square. What is the maximum number of elements that  $A$  can have?

**About the PEM Weekly Problems**

The PEM Weekly Problems aims to challenge and enrich high school students' creativity and critical thinking skills by exposing them to non-routine math problems and puzzles. While the problem sets are primarily intended for PEM participants, everyone is encouraged to submit their solutions to us. We acknowledge on the page everyone who submits correct answers. Moreover, PEM participants who solve the most number of problems will be recognized and awarded during the PEM closing ceremony.

**Submitting Solutions**

1. Typeset and handwritten solutions are welcome. For handwritten solutions, please scan or take a clear photo of your paper.
2. Indicate in the submission your name, school, and year level.
3. Send your solution to [ateneo.tuklas@gmail.com](mailto:ateneo.tuklas@gmail.com).