

Sociable Numbers

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Posted October 1, 2016 at www.ateneo.edu/ls/sose/mathematics/tuklas

The number 6 has an interesting property. Its factors are 1, 2, 3, and 6. If we add all the factors of 6 that are not equal to itself, then we get $1 + 2 + 3 = 6$, which is equal to itself – a happy coincidence! The number 6 is called a *perfect number*.

Now if we take the number 220, and add all its factors that are not equal to itself, we would get 284. If we take 284 and do to it what we did for 220, we would get 220 – another happy coincidence! In this case, the numbers 220 and 284 are called *amicable numbers*.

From these two observations, we begin to wonder: are there more perfect numbers or amicable numbers, and are there numbers with similar properties? To aid them in their search, mathematicians have created a function called a divisor function $s(x)$ (this is one of the many divisor functions out there), which gives the sum of all proper divisors of a number; i.e. the sum of all divisors of a number except itself. As an example, $s(6) = 6$, $s(220) = 284$, $s(284) = 220$, and $s(0)$ is not defined because 0 has no divisors.

From here, we can repeatedly apply the divisor function to a number n . What we get from this is a sequence of numbers $n, s(n), s(s(n)), \dots$. This sequence is called the aliquot sequence of the number n . For example, the aliquot sequence of 8 is 8, 7, 1, 0. In this example, we stop at 0 because 0 has no divisors.

Interestingly, it is possible that after writing down some terms of the aliquot sequence of a number n , one may find out that the n appears again in the sequence; i.e. the sequence cycles back to n . If that is the case, then we call n a *sociable number of order k* , where k is the number of terms in the aliquot sequence that come before the second appearance of n . For example, the amicable number 220 is a sociable number of order 2 because its aliquot sequence is 220, 284, 220, 284, \dots ; 2 numbers come before the second appearance of 220. We can also infer from here that perfect numbers are sociable numbers of order 1 because the sum of the proper divisors of a perfect number is itself.

Some questions would naturally arise: are there sociable numbers of orders other than 1 or 2? How many perfect numbers and amicable numbers are there? The answer to the first is yes: mathematicians have found sociable numbers of orders 1, 2, 4, 5, 6, 8, 9, and 28; however, they have yet to find sociable numbers of orders other than those mentioned. The answer to the second, on the other hand, continues to elude mathematicians. In fact, mathematicians have yet to find out whether there is a biggest perfect number (amicable number resp.), and they have yet to determine whether there exists a formula that would give all perfect numbers or amicable numbers!

The property of “sociability” is one of the interesting properties that whole numbers have, and it is one that has fascinated mathematicians around the world for centuries. While simple and readily accessible to the general populace – one needs to know only addition and prime factorization to start finding sociable numbers – these problems proved to have stood the test of time, and likely will elude mathematical methods for a long time.



References

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